SUBIECTUL I

ECUATII DIFERENTIALE DE ORDINUL I , explicite in y'

NR	DENUMIRE	FORMA	SCHIMBAREA DE VARIABILA / REZOLVARE	CE DEVINE
1	Ecuatii cu variabile separabile	$x' = g(t) \cdot h(x)$	$\int \frac{x'}{h(x)} dt = \int g(t)dt$	
2	Ecuatii omogene	$x' = h(\frac{x}{t})$	$\frac{x}{t} = y$, x = yt, $x' = y' t + y$	ecuatie cu variabile separabile
3	Ecuatii reductibile la cele omogene	$x' = f\left(\frac{a_1x+b_1t+c_1}{a_2x+b_2t+c_2}\right)$	$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$, rezolv sist $\begin{cases} a_1x + b_1t + c_1 = 0 \\ a_2x + b_2t + c_2 = 0 \end{cases}$ $\Rightarrow (x_0 , y_0)$ solutie $\begin{matrix} SV \\ SF \end{matrix} \begin{cases} \tau = t - t_0 \\ y = x - x_0 \end{cases}$	ecuatie omogena
			$\Delta = 0 \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = k$ SF $y = a_2 x + b_2 t$	ecuatie cu variabile separabile
4	Ecuatii liniare de ordinul I	$x' = P(t)x + Q(t)$	1.Ecuatia omogena: $x' = P(t)x$, se integreaza $\Rightarrow$ solutia generala $y = C_1 g(t)$ 2.Se cauta o solutie particulara a ecuatiei neomogene cu metoda variatiei constantelor: $Y_P = C_1(t) g(t)$, unde C_1 se determina prin inlocuirea in ecuatia neomogena. 3. Solutia generala este $X_{gen\ neomogena} = x_{gen\ omogena} + x_{P\ neom}$	
5	Ecuatii Bernoulli	$x' = P(t)x + Q(t)x^{\alpha}$ $\alpha \in R \setminus \{0,1\}$	SF: $y = x^{1-\alpha}$	ecuatie liniara
6	Ecuatii Riccati	$x' = a(t)x^2 + b(t)x + c(t)$ cu sol partic x_p	SF : $y = x - x_p$	ecuatie Bernoulli

OBS: Se da o ecuatie omogena si o ecuatie liniara sau Bernoulli

SUBIECTUL 2

ECUATII CU DIFERENTIALE EXACTE. ECUATII CU FACTOR INTEGRANT.

$P(x,t)dx + Q(x.t)dt = 0$

1) Daca $\frac{\partial P}{\partial t} = \frac{\partial Q}{\partial x}$, exista F asa incat dF = Pdx + Qdt si solutia ecuatiei este F(x,t) = 0

F se determina rezolvand sistemul $\begin{cases} \frac{\partial F}{\partial x} = P \\ \frac{\partial F}{\partial t} = Q \end{cases}$, prin integrarea primei ecuatii in raport cu x si inlocuirea lui F(x,t) = $\int P(x,t)dx + C(t)$ in a doua ecuatie.

Exista si formula directa de calcul pentru F:

F(x,t) = $\int_{x_0}^{x} P(\tau, t_0)d\tau + \int_{t_0}^{t} Q(x,\tau)d\tau$.

2) $\frac{\partial P}{\partial t} \neq \frac{\partial Q}{\partial x}$, cautam factor integrant:

a) Daca $\frac{1}{Q}\left(\frac{\partial P}{\partial t} - \frac{\partial Q}{\partial x}\right) = \varphi(t)$, luam ρ = ρ(t), solutie a ecuatiei

ρ'$= \varphi(t)$ρ

b) Daca $\frac{1}{P}\left(\frac{\partial P}{\partial t} - \frac{\partial Q}{\partial x}\right) = \psi(x)$, luam ρ = ρ(x), solutie a ecuatiei

ρ'$= \psi(x)$ρ

Inmultim ecuatia cu ρ si obtinem o ecuatie cu diferentiala exacta pe care o integram.

SUBIECTUL 3

ECUATII DIFERENTIALE LINIARE DE ORDIN SUPERIOR CU COEFICIENTI CONSTANTI

$$a_0x^{(n)} + a_1x^{(n-1)} + \cdots + a_{n-1}x' + a_nx = f(t)$$

1) Asociez **ecuatia omogena**

$a_0x^{(n)} + a_1x^{(n-1)} + \cdots + a_{n-1}x' + a_nx = 0$, cu ecuatia caracteristica asociata $a_0r^n + a_1r^{n-1} + \cdots + a_{n-1}r + a_n = 0$, cu solutiile r_1, r_2, … , r_n . Solutia generala a ecuatiei omogene va fi o suma de expresii de forma:

a) C_1 e^{r_it} ,daca r_i este radacina reala simpla

b) ($C_1 + C_2t + … + C_p t^{p-1}$) e^{r_it}, daca r_i este radacina reala multipla de ordin p

c) C_1e^{at} cos bt + C_2e^{at} sin bt , daca avem radacinile comlpexe conjugate simple r_1 = a + ib si r_2 = a – ib.

d) $(C_1 + C_2t + \dots + C_p t^{p-1}) e^{at} \cos bt + (D_1 + D_2t + \dots + D_p t^{p-1}) e^{at} \sin bt$, daca avem radacinile comlpexe conjugate $r_1 = a + ib$ si $r_2 = a - ib$, multiple de ordin p.

2) Determinam o **solutie particulara** a ecuatiei neomogene, dupa forma termenului liber (f(t)):

a) $f(t) = P_k(t)$ = polinom de grad k , aleg

a_1) $x_p = Q_k(t)$ = polinom de grad k in t cu coeficienti arbitrari, daca 0 nu este radacina a ecuatiei caracteristice.

a_2) $x_p = t^m Q_k(t)$, cu $Q_k(t)$ = polinom de grad k in t cu coeficienti arbitrari, daca 0 este radacina multipla de ordin m a ecuatiei caracteristice .

b) $f(t) = a e^{bt}$, aleg

b_1) $x_P = A e^{bt}$,daca b nu este radacina a ecuatiei caracteristice.

b_2) $x_P = At^m e^{bt}$,daca b este radacina multipla de ordin m a ecuatiei caracteristice.

c) $F(t) = a_1 \cos bt + a_2 \sin bt$, aleg

c_1) $x_p = A \cos bt + B \sin bt$, daca bi nu este radacina a ecuatiei caracteristice.

c_2) $x_p = t^m(A \cos bt + B \sin bt)$, daca bi este radacina multipla de ordin m a ecuatiei caracteristice.

Obs. Se poate folosi si metoda variatiei constantelor.

SUBIECTUL 4

ECUATII DIFERENTIALE CU ERIVATE PARTIALE DE ORDINUL I

1) LINIARE

$X_1 \frac{\partial u}{\partial x_1} + X_2 \frac{\partial u}{\partial x_2} + \dots + X_n \frac{\partial u}{\partial x_n} = 0$, unde u este functia necunoscuta.

a) Asociez sistemul caracteristic : $\frac{dx_1}{X_1} = \frac{dx_2}{X_2} = \cdots = \frac{dx_n}{X_n}$

b) Integrez acest sistem determinand n-1 integrale prime functional independente, cu urmatoarele metode:

b_1) Daca doua fractii contin numai doua variabile, se ia deoparte egalitatea lor si se integreaza ca o ecuatie diferentiala obisnuita.

b_2) Daca am obtinut deja o integrala prima se scoate o variabila in functie de celelalte variabile si de constanta C_1, din aceasta integrala prima si se inlocuiesta in sistemul caracteristic, pentru ca sa obtinem doua fractii in numai doua necunoscute.

b_3) Aplicand proportiile derivate, punem in evidenta expresii de forma $\frac{du}{u}$, care se egaleaza cu alte fractii de acelasi tip, pentru ca sa obtinem doua fractii in numai doua necunoscute.

Obtinem sistemul

$F_1(x_1, x_2, \dots, x_n) = C_1$

$F_2(x_1, x_2, \dots, x_n) = C_2$

…………………………………………

$F_{n-1}(x_1, x_2, \dots, x_n) = C_{n-1}$

Solutia generala este

$u(x_1, x_2, \dots, x_n) = \phi(F_1(x_1, x_2, \dots, x_n), \dots, F_{n-1}(x_1, x_2, \dots, x_n))$, $\phi \in C^2$ oarecare.

2) CVASILINIARE

$X_1 \frac{\partial u}{\partial x_1} + X_2 \frac{\partial u}{\partial x_2} + \dots + X_n \frac{\partial u}{\partial x_n} = X_{n+1}$

c) Asociez sistemul caracteristic : $\frac{dx_1}{X_1} = \frac{dx_2}{X_2} = \cdots = \frac{dx_n}{X_n} = \frac{du}{X_{n+1}}$

d) Integrez acest sistem determinand n integrale prime functional independente:

$F_1(x_1, x_2, \dots, x_n, u) = C_1$

$F_2(x_1, x_2, \dots, x_n, u) = C_2$

…………………………………………

$F_n(x_1, x_2, \dots, x_n, u) = C_n$

Solutia generala este $\phi(F_1(x_1, x_2, \dots, x_n, u), \dots, F_{n-1}(x_1, x_2, \dots, x_n, u)) = 0$.

SUBIECTUL 5

ECUATII DIFERENTIALE CU ERIVATE PARTIALE DE ORDINUL II

$$A(x,y) \frac{\partial^2 u}{\partial x^2} + 2B(x,y) \frac{\partial^2 u}{\partial x \partial y} + C(x,y) \frac{\partial^2 u}{\partial y^2} + D\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, u, x, y\right) = 0$$

1) Asociez ecuatia proiectiilor curbelor caracteristice

$$A(x,y) \left(\frac{dy}{dx}\right)^2 - 2B(x,y) \frac{dy}{dx} + C(x,y) = 0$$

Notam $t = \frac{dy}{dx}$ si rezolvam ecuatia obtinuta $At^2 - 2Bt + C = 0$

a) $\Delta > 0 \Leftrightarrow B^2 - AC > 0 \Rightarrow$ ecuatie de tip **hiperbolic**

Ecuatia in t are doua radacini reale distincte t_1 si t_2.

$\frac{dy}{dx} = t_1 \Rightarrow \varphi_1(x,y) = C_1$

$\frac{dy}{dx} = t_2 \Rightarrow \varphi_2(x,y) = C_2$

Facem schimbarea e functii $\begin{cases} \xi = \varphi_1(x,y) \\ \eta = \varphi_2(x,y) \end{cases}$ si obtinem ecuatia canonica

$$\frac{\partial^2 U}{\partial \xi^2} = \Phi_1 \left(\frac{\partial U}{\partial \xi}, \frac{\partial U}{\partial \eta}, U, \xi, \eta\right)$$

b) $\Delta = 0 \Leftrightarrow B^2 - AC = 0 \Rightarrow$ ecuatie de tip **parabolic**

Ecuatia in t are doua radacini reale egale $t_1 = t_2$

$\frac{dy}{dx} = t_1 \Rightarrow \varphi_1(x,y) = C_1$

Facem schimbarea e functii $\begin{cases} \xi = \varphi_1(x,y) \\ \eta = x \end{cases}$ si obtinem ecuatia canonica

$$\frac{\partial^2 U}{\partial \eta^2} = \Phi_2 \left(\frac{\partial U}{\partial \xi}, \frac{\partial U}{\partial \eta}, U, \xi, \eta\right)$$

c) $\Delta < 0 \Leftrightarrow B^2 - AC < 0 \Rightarrow$ ecuatie de tip **eliptic**

Ecuatia in t are doua radacini complexe conjugate t_1 si t_2

$\frac{dy}{dx} = t_1 \Rightarrow \varphi_1(x,y) = C_1$

$\frac{dy}{dx} = t_2 \Rightarrow \varphi_2(x,y) = C_2$

Facem schimbarea e functii $\begin{cases} \xi = \frac{\varphi_1 + \varphi_2}{2} \\ \eta = \frac{\varphi_1 - \varphi_2}{2} \end{cases}$ si obtinem ecuatia canonica

$$\frac{\partial^2 U}{\partial \xi^2} + \frac{\partial^2 U}{\partial \eta^2} = \Phi_3 \left(\frac{\partial U}{\partial \xi}, \frac{\partial U}{\partial \eta}, U, \xi, \eta\right).$$

OBS. Daca A , B si C sunt constante , atunci toate $\Phi_i = 0$, i = 1,2,3 si ecuatia se poate integra usor.